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ILSCR rumor spreading model to discuss the control of rumor spreading in emergency

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HIGHLIGHTS

- A kinetic model with cooling off period for the spread of rumor is considered.
- The spreaders may experience a cooling off period before becoming stiflers.
- The mobility of people in a certain area is considered in the proposed model.
- The implications of the results are discussed with a chemical explosion rumor event.
- Some advice for emergency responders are proposed by us to curb the spread of rumor.

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ABSTRACT

Destructive rumors can infringe upon others' interests, disrupt public order and pose a threat to social stability. For crisis communicators, it is important to understand when and how rumor should be controlled. In this paper, taking into account the possibility of that the spreaders may experience a cooling off period before becoming stiflers and the mobility of people in a certain area, we propose a novel rumor spreading model to discuss the control of rumor spreading in emergency. We also analyze the stability of the proposed model. The validity of the obtained theoretical results is verified by numerical results. Numerical simulations are performed to investigate the impact of rumor control measures on the spread of rumor. Finally, we suggest means through which crisis management departments can curb the spread of rumor.

1. Introduction

For crisis communicators, it is important to understand when and how rumor should be controlled. Emergencies such as hurricane and chemical pollution typically trigger the spread of rumor, which is a mathematically interesting and socially important phenomenon [1–6]. Rumors represent information on topics of public interest that spread through social networks but that may be based on inaccurate descriptions of problems or events [7]. Destructive rumors can infringe upon others' interests, disrupt public order and pose a threat to social stability [8–10]. Often, rumors affect how rational individuals assess risks, evaluate needs, and make decisions in disaster-affected environments [11]. On Feb 10, 2011, for example, a blast rumor triggered a fatal stampede in Jiangsu Province, China. More than 10,000 residents of the coastal county named Xiangshui abruptly fled their homes after a rumor surfaced that a chemical plant in the Chenjiagang industrial park was leaking chlorine and an ensuing explosion could occur. In the chaotic scene, a motor tricycle with more than 20 people onboard crashed into a river, resulting in the deaths of four. A few relatively minor accidents also occurred in the rainy and snowy weather. This incident aroused widespread social concern in China.

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Understanding the spreading characteristics of rumor can lead to better approaches to suppressing the spread of rumor [12]. Mathematical models have become important tools in analyzing the spread and control of rumors [2,4,13–16]. Rumors can be viewed as an "infection of the mind", and their spreading shows an interesting resemblance to that of epidemics. Pioneering contributions to their modeling, based on epidemic models, date back to Daley [17]. Afterwards, a few variants of these two classical rumor spreading models are formulated and applied [12,18–22]. Daley [17] introduces a classical rumor spreading model. In their model, a closed and homogeneously mixed population is subdivided into three groups: those who are ignorant of the rumor (ignorants), those who have heard it and actively spread it (spreaders), and those who are aware of the rumor but choose not to spread it (stiflers). The authors assume that the rumor is propagated through the population by pair-wise contacts between spreaders and others in the population. When a spreader contacts an ignorant, in this case, the latter becomes a spreader; in the other two cases, either one or both of those involved in the meeting learn that the rumor is 'known' and choose not to spread the rumor anymore, thereby turning into stiflers. Kawachi [23] considers a rumor spreading model with various contact interactions and explore what effect such interactions have on the spread of a rumor, in particular whether they can explain the rumor recursion. The author takes into consideration the possibility of the transition from the stifler class into the susceptible class when treating a variable rumor. Belen et al. [18] refine the classical MT model in continuous time by considering general initial conditions. In their study, the classical fourth order Runge–Kutta method is applied to estimate the error of the proposed model. Komi [24] develops a rumor spreading model by considering the forgetting mechanism and population's education rate. In their study, they consider that the educated ignorant individuals have less contribution into the spreader class but more contribution into the stifler class than the non-educated ignorant individuals.

The development of complex network theory makes the study of rumor spreading step into a new era [6,20,25–27]. Doer et al. [6] analyze how rumors spread in social networks and simulate a rumor spreading process in different network topologies. They perform a mathematical analysis of this process in preferential attachment graphs and observe that nodes with few neighbors are crucial for the fast dissemination. Moreno et al. [20] use mean-field equations to describe the dynamics of the model on complex social networks. The authors introduce a stochastic method that allows us to obtain meaningful time profiles for the quantities characterizing the propagation process. Zhu and Zhao [25] propose a model with discrete and nonlocal delays for investigating the Spatial–temporal dynamics of rumor spreading in online social networks. The authors assume that a time delay exists before the authorities' actions on rumor diffusion. Ma et al. [26] investigate the effect of bipolar social reinforcement on the rumor spreading dynamics in the online social network. They use the generation function and cavity method developed from statistical physics of disordered system to calculate the threshold for rumor spreading. Liu et al. [27] study a SEIR rumor propagation model on heterogeneous network by considering that the exposed nodes may become the removed nodes at a rate.

The above rumor spreading models have two pities. First, almost all of the existing literature on rumor spreading model assumes that the spreaders directly become stiflers. However, this assumption ignores the possibility that the spreaders may experience a cooling off period before becoming stiflers. For convenience, those who belong to the cooling off class are called cooled. This possibility lies in that, from the spreaders' perspective, in the early stages of an emergency, if the institutional mainstream media has a lower credibility, some spreaders may initially tend to trust information they receive from their acquaintances such as relatives, friends, and neighbors instead of trusting institutional mainstream media. When the acquaintances who think the rumor is fabricated contact the spreaders, the spreaders may begin to gradually reduce the external dissemination of the rumor, in this case, the spreader turned into the cooled. Later, the cooled find that there is no chemical explosion there and confirm that the rumor is fabricated, and they may cease to spread the rumor to others, in this case, the cooled became the stifler. In this paper, from the spreaders' perspective, this process is referred to as a cooling off period. Based on these observations, it might seem plausible that we extend existing rumor spreading model by considering the situation of that a certain percentage of the spreaders may experience a cooling off period before becoming stiflers.

Second, another pity of the above rumor spreading mechanisms is that the mobility of the population is not considered in most of existing rumor spreading models. In other words, there is no outflow from any of the classes or inflow to the susceptible class in their models. Still, their models are extremely innovative and very useful in the modeling and analysis of rumor spreading.

In order to make the rumor spreading process fit realistic cases more closely, in this paper, taking into account the possibility of that the spreaders may experience a cooling off period before becoming stiflers and the mobility of people in a certain area, we propose a novel rumor spreading model called the ILSCR model to discuss the control of rumor spreading in emergencies.

The rest of this paper is organized as follows: In Section 2, we describe our ILSCR rumor spreading model. In Section 3, we conduct a stability analysis of the proposed model. In Section 4, we present the numerical results followed by a discussion of the implications of these simulation results. Finally, we draw some conclusions, discuss some limitations and propose some future research directions in Section 5.

2. The ILSCR rumor spreading model

In this section, we describe our ILSCR rumor spreading model. Let $N(t)$ denote the total population at time t. As shown in Fig. 1, the population is divided into five groups: those who are ignorant of the rumor (Ignorants), those who could not confirm the information immediately after they heard the rumor (Latents), those who know the rumor and spread it actively

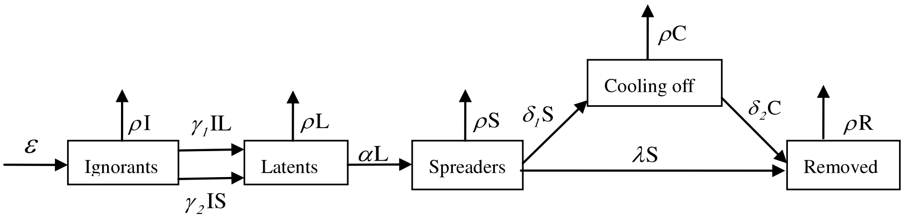

Fig. 1. ILSCR rumor spreading process.

(Spreaders), those who initially distrust the institutional mainstream media but turn to their acquaintances to obtain crisis information that is needed for their understanding of the local situation and decision making, thereafter, gradually reduce the external dissemination of the rumor after their acquaintances, who think the rumor is fabricated, contact them, finally, cease to spread the rumor to the others after they confirm that the rumor is fabricated (Cooled), and those who already know the rumor but choose not to spread it (Removed). For convenience, those who belong to the removed class are called stiflers. We denote the number of ignorants, latents, spreaders, cooled and stiflers at time t as $I(t)$, $L(t)$, $S(t)$, $C(t)$ and $R(t)$, respectively. Then we have

$$N(t) = I(t) + L(t) + S(t) + C(t) + R(t)$$

The process of ILSCR rumor spreading model is shown in Fig. 1.

First, we consider the spread of rumor with variable population size and assume that the rumor spreads in a population with constant immigration and emigration. All recruitment is into the ignorant class and occurs at a positive constant ε, and ρ is the emigration rate of those five class, respectively.

As shown in Fig. 1, the rumor spreading rules of our ILSCR model can be summarized as follows.
(I) When an ignorant contacts a latent, the former turns into a new latent with probability γ_1.
(II) When an ignorant contacts a spreader, the former becomes a latent with probability γ_2.
(III) Some latents may tend to believe the rumor and spread it to the ignorants, in this case, the latents turn into spreaders with probability α.
(IV) After being informed of the truth by the institutional mainstream media, some spreaders may tend to trust institutional mainstream media and cease to spread the rumor to others, in this case, the spreaders change into stiflers with probability λ; this is known as the stifling rate.
(V) From the spreaders' perspective, in the early stages of an emergency, if the institutional mainstream media has a lower credibility, some spreaders may tend to trust information they receive from their acquaintances instead of trusting institutional mainstream media. When the acquaintance who think the rumor is false contacts a spreader, and the latter may begin to gradually reduce the external dissemination of the rumor, in this case, the spreader turns into a cooled with probability δ_1; this is known as the cooling off rate.
(VI) When the cooled eventually realizes the rumor is completely fabricated and ceases to spread the rumor to the others, in this case, the cooled becomes a stifler with probability δ_2.

The main difference between the cooled and the removed is that: the cooled still have 'infectious force', and their 'infectious force' is weaker than that of the spreaders. However, the removed do not have any 'infectious force'. In other words, the cooled gradually reduce the external dissemination of the rumor after being informed by their acquaintances who think that the rumor is fabricated, however, the removed have completely abandoned the spread of rumor. By caring about the cooled group, we want to highlight the credibility of the mainstream media may have an important influence on the spread of rumors in emergencies.

According to the ILSCR rumor spreading process elaborated above, the differential equations can be described as follows:

$$\begin{cases} \dfrac{dI(t)}{dt} = \varepsilon - \gamma_1 I(t)L(t) - \gamma_2 I(t)S(t) - \rho I(t), \\ \dfrac{dL(t)}{dt} = \gamma_1 I(t)L(t) + \gamma_2 I(t)S(t) - (\alpha + \rho)L(t), \\ \dfrac{dS(t)}{dt} = \alpha L(t) - (\delta_1 + \lambda + \rho)S(t), \\ \dfrac{dC(t)}{dt} = \delta_1 S(t) - (\delta_2 + \rho)C(t), \\ \dfrac{dR(t)}{dt} = \lambda S(t) + \delta_2 C(t) - \rho R(t). \end{cases} \tag{1}$$

From (1), we know

$$\frac{dN(t)}{dt} = \frac{dI}{dt} + \frac{dL}{dt} + \frac{dS}{dt} + \frac{dC}{dt} + \frac{dR}{dt} = \varepsilon - \rho N(t). \tag{2}$$

The positive invariant set of system (1) is:

$$\Omega = \{(I, L, S, C, R) \in \mathbb{R}_+^5 : 0 \le I + L + S + C + R \le \frac{\varepsilon}{\rho}\}. \tag{3}$$

It is easy to observe that the model has a rumor-free equilibrium point given by the following: $\phi_0 = (\frac{\varepsilon}{\rho}, 0, 0, 0, 0)$.
To find the rumor-prevailing equilibrium $\phi^* = (I^*, L^*, S^*, C^*, R^*)$, set

$$\begin{cases} \varepsilon - \gamma_1 I^* L^* - \gamma_2 I^* S^* - \rho I^* = 0, \\ \gamma_1 I^* L^* + \gamma_2 I^* S^* - (\alpha + \rho) L^* = 0, \\ \alpha L^* - (\delta_1 + \lambda + \rho) S^* = 0, \\ \delta_1 S^* - (\delta_2 + \rho) C^* = 0, \\ \lambda S^* + \delta_2 C^* - \rho R^* = 0. \end{cases} \tag{4}$$

By some simple calculation, we can show that a unique ϕ^* exists with

$$\begin{aligned} I^* &= \frac{\varepsilon}{\rho} - \frac{(\alpha+\rho)(\delta_1+\lambda+\rho)}{\rho\alpha} S^*, \qquad L^* = \frac{\delta_1+\lambda+\rho}{\alpha} S^*, \\ C^* &= \frac{\delta_1}{\delta_2+\rho} S^*, \qquad R^* = \frac{\lambda(\rho+\delta_2)+\delta_1\delta_2}{\rho(\rho+\delta_2)} S^*, \\ S^* &= \frac{\alpha\varepsilon}{(\delta_1+\lambda+\rho)(\rho+\alpha)} - \frac{\rho\alpha}{\gamma_1(\delta_1+\lambda+\rho)+\gamma_2\alpha}. \end{aligned} \tag{5}$$

Then, we will find the basic reproduction number R_0 of system (1) by the next generation matrix [28]:

$$R_0 = \frac{\gamma_2\alpha\varepsilon}{\rho(\alpha+\rho)(\delta_1+\lambda+\rho)} + \frac{\varepsilon\gamma_1}{\rho(\alpha+\rho)}. \tag{6}$$

Here, R_0 is defined as the expected number of a new generation of rumor spreaders produced by a single spreader. According to [28], we know that R_0 is an important parameter which determines the number of equilibrium.

Next, we consider the global stability of ϕ_0 and ϕ^*, respectively.

3. Stability analysis of ILSCR rumor spreading model

Theorem 1. *If $R_0 < 1$, the rumor-elimination equilibrium ϕ_0 of Eq. (1) is locally asymptotically stable.*

Proof. The Jacobian matrix of Eq. (1) at $\phi_0 = (\frac{\varepsilon}{\rho}, 0, 0, 0, 0)$ is

$$J(\phi_0) = \begin{bmatrix} -\rho & -\frac{\gamma_1\varepsilon}{\rho} & -\frac{\gamma_2\varepsilon}{\rho} & 0 & 0 \\ 0 & \frac{\gamma_1\varepsilon}{\rho} - \rho - \alpha & \frac{\gamma_2\varepsilon}{\rho} & 0 & 0 \\ 0 & \alpha & -\delta_1 - \rho - \lambda & 0 & 0 \\ 0 & 0 & \delta_1 & -\rho - \delta_2 & 0 \\ 0 & 0 & \lambda & \delta_2 & -\rho \end{bmatrix}. \tag{7}$$

Then, we can get that $J(\phi_0)$ has always three negative eigenvalues $\lambda_{1,2} = -\rho < 0$, $\lambda_3 = -\rho - \delta_2 < 0$. The other two eigenvalues λ_4, λ_5 are eigenvalues of the 2×2 matrix

$$\Gamma = \begin{bmatrix} \frac{\gamma_1\varepsilon}{\rho} - \alpha - \rho & \frac{\gamma_2\varepsilon}{\rho} \\ \alpha & -\delta_1 - \lambda - \rho \end{bmatrix}. \tag{8}$$

We want to show, when $R_0 < 1$, that Routh–Hurwitz conditions hold, namely, $\mathrm{tr}(\Gamma) < 0$ and $\det(\Gamma) > 0$.
From Eq. (8), we obtain

$$\mathrm{tr}(\Gamma) = \frac{\gamma_1\varepsilon}{\rho} - \alpha - \lambda - \delta_1 - 2\rho. \tag{9}$$

Because $R_0 < 1$, we have

$$\gamma_2\alpha\varepsilon + \varepsilon\gamma_1(\delta_1+\lambda+\rho) < \rho(\alpha+\rho)(\delta_1+\lambda+\rho). \tag{10}$$

Hence, we know

$$\frac{\gamma_1\varepsilon}{\rho} < \alpha + \rho. \tag{11}$$

From Eqs. (9) and (11), we have

$$\mathrm{tr}(\Gamma) < 0. \tag{12}$$

Then

$$\det(\Gamma) = \frac{\rho(\alpha+\rho)(\delta_1+\beta+\rho) - \gamma_1\varepsilon(\delta_1+\beta+\rho) - \gamma_2\alpha\varepsilon}{\rho}. \tag{13}$$

From Eqs. (10) and (13) , we can get

$$\det(\Gamma) > 0, \quad \text{if and only if} \quad R_0 < 1. \tag{14}$$

By Eqs. (12) and (14), we know that $\lambda_4 < 0$ and $\lambda_5 < 0$. Hence, for Eq. (1), by Routh–Hurwitz criterion [28], the rumor-elimination equilibrium ϕ_0 is locally asymptotically stable if $R_0 < 1$, completing the proof.

In control system theory, the Routh–Hurwitz criterion is a mathematical test that is a necessary and sufficient condition for the stability of a time invariant control system [28]. The importance of the criterion is that all the roots h of the characteristic equation of a linear system with negative real parts represent solutions $exp(ht)$ of the system that are stable [29]. The interested reader should consult references, such as Ma [28] and Routh [29], for details.

Next, we focus on investigating the global stability of the rumor-elimination equilibrium.

Theorem 2. *When $R_0 < 1$, the rumor-elimination equilibrium ϕ_0 of Eq. (1) is globally asymptotically stable in the feasible region Ω.*

Proof. We take a Lyapunov function $V(L, S)$ of form

$$V(L,S) = L + \frac{\alpha+\rho}{\alpha}S. \tag{15}$$

By Eqs. (1) and (15), we have

$$\begin{aligned}
\frac{dV}{dt} &= \frac{dL}{dt} + \frac{\alpha+\rho}{\alpha}\frac{dS}{dt} \\
&= \gamma_1 IL + \gamma_2 IS - (\alpha+\rho)L + \frac{\alpha+\rho}{\alpha}(\alpha L - (\delta_1+\lambda+\rho)S) \\
&= \gamma_1 I\frac{\delta_1+\lambda+\rho}{\alpha}S + \gamma_2 IS - (\alpha+\rho)L + \frac{\alpha+\rho}{\alpha}(\alpha L - (\delta_1+\lambda+\rho)S) \\
&\le (\gamma_1\frac{\varepsilon}{\rho}\frac{\delta_1+\lambda+\rho}{\alpha} + \gamma_2\frac{\varepsilon}{\rho} - \frac{(\alpha+\rho)(\delta_1+\lambda+\rho)}{\alpha})S \\
&= (R_0 - 1)\frac{(\alpha+\rho)(\delta_1+\lambda+\rho)}{\alpha}S \le 0.
\end{aligned} \tag{16}$$

So $\frac{dV}{dt} = 0$ if and only if $S = 0$. By LaSalle's Invariance Principle [30], ϕ_0 is globally asymptotically stable in Ω, completing the proof.

The LaSalle Invariance Principle provides a generalization of Lyapunov criteria for asymptotic stability. The idea of the LaSalle Invariance Principle is that, if in a domain about the origin, we can find a Lyapunov function whose derivative along the trajectories of the system is negative semi-definite, and we can further establish that no trajectory can stay identically at points where $\frac{dV}{dt} = 0$, except at the origin, then the origin is asymptotically stable [30,31].

Next, we focus on investigating the global stability of the rumor-prevailing equilibrium ϕ^* when $R_0 > 1$.

Once the dynamics of (I, L, S) are understood, those of C, R can then be determined from the last two equations of Eq. (1). Then we consider the sub-system of Eq. (1)

$$\begin{cases}
\frac{dI(t)}{dt} = \varepsilon - \gamma_1 I(t)L(t) - \gamma_2 I(t)S(t) - \rho I(t), \\
\frac{dL(t)}{dt} = \gamma_1 I(t)L(t) + \gamma_2 I(t)S(t) - (\alpha+\rho)L(t), \\
\frac{dS(t)}{dt} = \alpha L(t) - (\delta_1+\lambda+\rho)S(t).
\end{cases} \tag{17}$$

This shows that the model can be studied in the feasible region

$$\Theta = \{(I,L,S) \in \mathbb{R}^3_+ : 0 \le I + L + S \le \frac{\varepsilon}{\rho}\}. \tag{18}$$

Then we can know that Eq. (17) has a positive equilibrium $\psi(I_*, L_*, S_*)$ when $R_0 > 1$.

Next we investigate the global stability of the rumor-prevailing equilibrium $\psi(I_*, L_*, S_*)$ of Eq. (17).

Theorem 3. *If $R_0 > 1$, then the rumor-prevailing equilibrium $\psi(I_*, L_*, S_*)$ of Eq. (17) is globally asymptotically stable.*

Proof. The Jacobian matrix of Eq. (17) at $\psi(I_*, L_*, S_*)$ is

$$G = \begin{bmatrix} -\gamma_1 L_* - \gamma_2 S_* - \rho & -\gamma_1 I_* & -\gamma_2 I_* \\ \gamma_1 L_* + \gamma_2 S_* & \gamma_1 I_* - \alpha - \rho & \gamma_2 I_* \\ 0 & \alpha & -\delta_1 - \lambda - \rho \end{bmatrix}. \tag{19}$$

Simple calculations show that

$$\mathrm{tr(G)} = -\frac{\varepsilon\alpha\gamma_2}{\alpha(\delta_1+\lambda+\rho)} - \rho - (\delta_1+\lambda+\rho) - \varepsilon(1-\frac{1}{R_0}) < 0. \tag{20}$$

$$\begin{aligned} \mathrm{det(G)} &= (-\gamma_1 L_* - \gamma_2 S_* - \rho)(\gamma_1 I_* - \alpha - \rho)(-\delta_1 - \lambda - \rho) - \gamma_2 I_* \alpha(\gamma_1 L_* + \gamma_2 S_*) + \\ &\quad \gamma_1 I_*(\gamma_1 L_* + \gamma_2 S_*)(-\delta_1 - \lambda - \rho) + \gamma_2 I_* \alpha(\gamma_1 L_* + \gamma_2 S_* + \rho) \\ &= -\varepsilon(1-\frac{1}{R_0})(\delta_1+\lambda+\rho)(\rho+\alpha) < 0. \end{aligned} \tag{21}$$

M denotes the sum of determinant of second-order principal minors of matrix G.

$$\begin{aligned} M &= -(\gamma_1 L_* + \gamma_2 S_* + \rho)(\gamma_1 I_* - \alpha - \rho) + \gamma_1 I_*(\gamma_1 L_* + \gamma_2 S_*) + \\ &\quad (\gamma_1 L_* + \gamma_2 S_* + \rho)(\delta_1+\lambda+\rho) + (\gamma_1 I_* - \alpha - \rho)(-\delta_1 - \lambda - \rho) - \gamma_2 I_* \alpha \\ &= (\varepsilon(1-\frac{1}{R_0}) + \rho)(\delta_1+\lambda+\rho) + \varepsilon(1-\frac{1}{R_0})(\rho+\alpha) - \frac{\varepsilon\gamma_1}{R_0}. \end{aligned} \tag{22}$$

Hence, according to Eqs. (20)–(22), we get

$$\mathrm{tr(G)}{*}\mathrm{M} - \mathrm{det(G)} < 0, \quad \text{if } R_0 > 1. \tag{23}$$

By Eqs. (20), (21), (23) and Routh–Hurwitz criterion, we know that all eigenvalues of Jacobian matrix G have negative real parts when R_0 is greater than one.

Next, we focus on investigating the global stability of rumor-prevailing equilibrium $\phi^* = (I^*, L^*, S^*, C^*, R^*)$ of Eq. (1).

Theorem 4. *If $R_0 > 1$, then the rumor-prevailing equilibrium $\phi^* = (I^*, L^*, S^*, C^*, R^*)$ of the Eq. (1) is globally asymptotically stable in Θ.*

Proof. From Theorem 3, we know that $(I, L, S) \to (I^*, L^*, S^*)$. Now we consider the sub-system of Eq. (1):

$$\begin{cases} \dfrac{dC(t)}{dt} = \delta_1 S(t) - (\delta_2+\rho)C(t), \\ \dfrac{dR(t)}{dt} = \lambda S(t) + \delta_2 C(t) - \rho R(t). \end{cases} \tag{24}$$

and its limit system is

$$\begin{cases} \dfrac{dC(t)}{dt} = \delta_1 S^*(t) - (\delta_2+\rho)C(t), \\ \dfrac{dR(t)}{dt} = \lambda S^*(t) + \delta_2 C(t) - \rho R(t). \end{cases} \tag{25}$$

Thus, we have

$$\begin{aligned} C(t) &= e^{-(\delta_2+\rho)t}[\delta_1 S^*(t)\int_0^t e^{(\delta_2+\rho)s} ds + C(0)], \\ R(t) &= e^{-\rho t}[\int_0^t (\lambda S^*(t) + \delta_2 C(t)) e^{\rho s} ds + R(0)]. \end{aligned}$$

Further, we can find that

$$\begin{aligned} C(t) &\to \frac{\delta_1 S^*}{\delta_2+\rho} = C^*, \\ R(t) &\to \frac{\lambda S^* + \delta_2 C^*}{\rho} = R^*, \quad t \to \infty. \end{aligned} \tag{26}$$

Hence, for system (1), the rumor-prevailing equilibrium point ϕ^* is globally asymptotically stable in Θ when R_0 is greater than one.

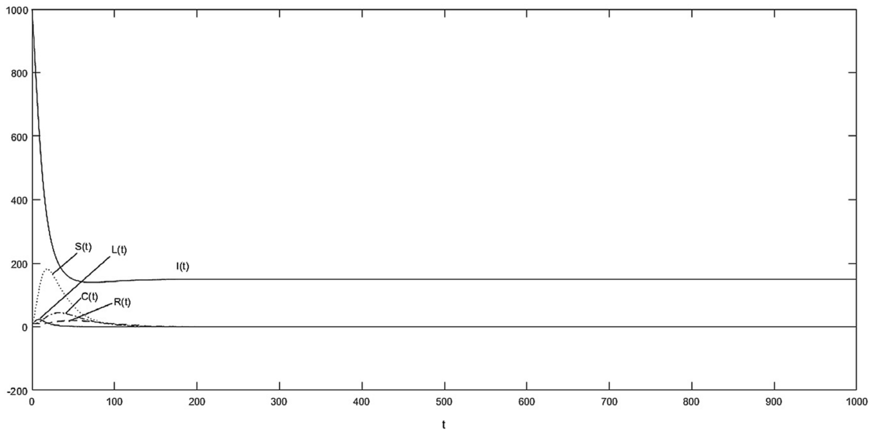

Fig. 2. Number of ignorants, latents, spreaders, cooled and stiflers over time t with $\varepsilon = 6$, $\rho = 0.04$, $\alpha = 0.9$, $\lambda = 0.001$, $\delta_1 = 0.02$, $\delta_2 = 0.02$, $\gamma_1 = 0.001$, $\gamma_2 = 0.0001$, $R_0 = 0.3950 < 1$.

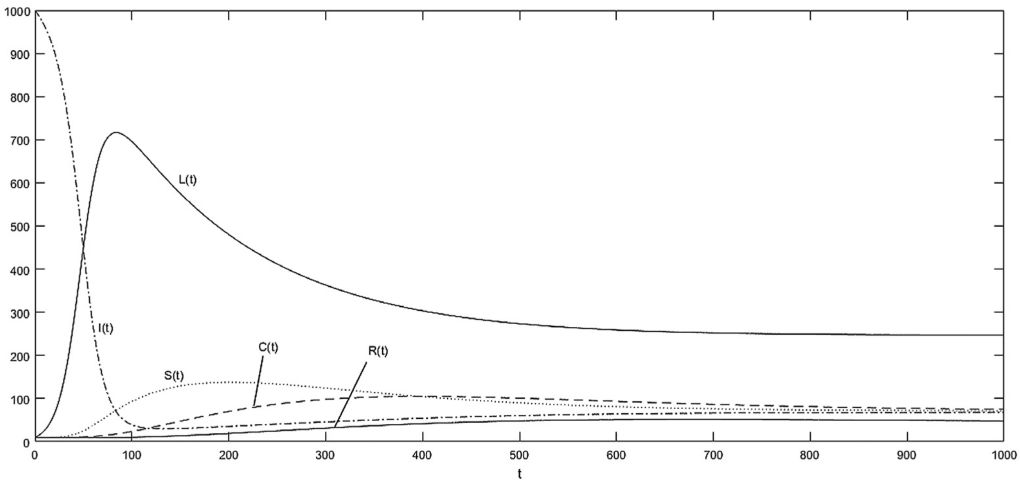

Fig. 3. Number of ignorants, latents, spreaders, cooled and stiflers over time t with $\varepsilon = 2$, $\rho = 0.004$, $\alpha = 0.003$, $\lambda = 0.04$, $\delta_1 = 0.006$, $\delta_2 = 0.02$, $\gamma_1 = 0.0001$, $\gamma_2 = 0.00001$, $R_0 = 7.1857 > 1$.

4. Numerical simulations and discussion

In this section, we perform numerical simulations to verify the theoretical results in previous sections and further investigate the impact of corresponding rumor control measures on the spread of rumor.

Firstly, Theorem 2 indicates that rumor dies out and rumor-free equilibrium is stable when R_0 is less than one. Fig. 2 further confirms this conclusion.

Fig. 2 shows how the number of the five kinds of agents changes over time t when R_0 is less than one.

From Fig. 2, we can see that the number of spreaders increases, reaches a peak, and then decreases to zero, at which point the rumor dies out. The variation trend of the number of cooled, stiflers and latents is similar to that of spreaders, respectively. However, the trend of increasing and decreasing processes of the stiflers is much more moderate than that of spreaders. The number of ignorants declines, reaches a minimum, then increases to a positive balanced value.

Secondly, Theorem 4 show that rumor is persists and stable when R_0 is greater than one. Fig. 3 further validates this conclusion.

Fig. 3 illustrates how the number of five kinds of agents changes over time t when R_0 is greater than one.

As can be seen in Fig. 3, the number of ignorants declines, reaches a minimum then increases to a positive balanced value. Throughout the entire process, the number of cooled consistently decreases until it reaches a stable value. The number of latents rapidly increases to a peak and then decreases until it reaches a positive stable value. The variation trend of the number of stiflers and spreaders is similar to that of latents, respectively. However, the trend of increasing and decreasing processes of spreaders is much more moderate than that of latents. It is worth noting that, the number of spreaders decreases

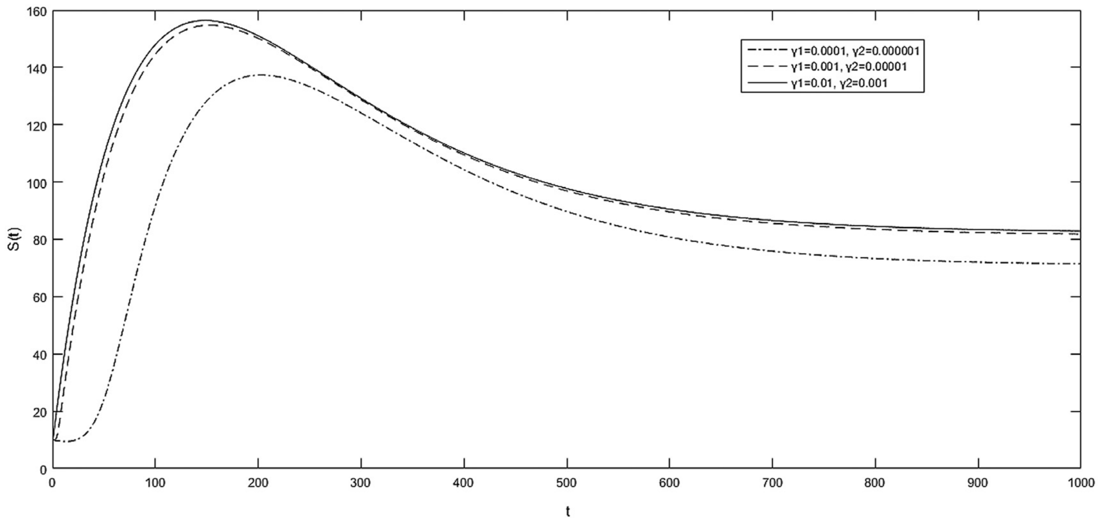

Fig. 4. Numbers of spreaders $S(t)$ versus time varying over different γ_1 and γ_2 with $\varepsilon = 2$, $\rho = 0.004$, $\alpha = 0.003$, $\delta_1 = 0.006$, $\lambda = 0.0004$, $\delta_2 = 0.02$.

to a positive stable value finally. This indicates that the rumor persists at a "prevalent" level. At this point the threshold parameter R_0 is greater than one.

Finally, we further investigate the impact of corresponding rumor control measures on the spread of rumor.

Recall the threshold parameter R_0 that we have

$$R_0 = \frac{\gamma_2 \alpha \varepsilon}{\rho(\alpha+\rho)(\delta_1+\lambda+\rho)} + \frac{\varepsilon \gamma_1}{\rho(\alpha+\rho)}. \tag{27}$$

From Eq. (27), we know that R_0 increases as γ_1 or γ_2 or ε increases, and decreases as δ_1 or λ increases. Moreover, if the stifling rate λ and the cooling off rate δ_1 are large enough, γ_1 and γ_2 are small enough such that R_0 is less than one, then the rumor dies out at this point. This means that the variables γ_1, γ_2, λ and δ_1 in the threshold parameter R_0 are four key parameters in the proposed model. Hence, to offer helpful suggestions to crisis management departments, we will discuss how the above four parameters affect the spread of rumor in our model.

First, we examine the effects of γ_1 and γ_2 on the spread of rumor.

Fig. 4 describes how the numbers of spreaders $S(t)$ change over time t under different γ_1 and γ_2.

From Fig. 4, we can see that the lower the value of γ_1 and γ_2, the smaller the number of spreaders. As shown in Fig. 4, the number of spreaders shows a modest decline when the value of γ_1 decreases from 0.01 to 0.0001 and that of γ_2 decreases from 0.001 to 0.00001. It should be noted that the decrease of the value of γ_1 and γ_2 primarily depends on enhancing the public's capacity of identifying rumors. One possible interpretation on this is that, if the public have stronger ability to identify rumors, such as they have more professional knowledge about chemical safety production, they will not blindly believe the rumor without prior confirmation and make irrational behavior. In the "2011 Xiangshui chemical explosion rumor" event, the local crisis management departments invited experts to provide the public with the relevant chemical safety knowledge, through multiple communication channels, after the rumor began to spread. Subsequently, many local residents on the streets realized that the chemical explosion rumor was false and started to return home. This suggests that enhancing the public's capacity of identifying rumors will be crucial to curbing rumors.

Fig. 5 demonstrates how the numbers of spreaders $S(t)$ change over time under different stifling rate λ.

As shown in Fig. 5, the number of spreaders shows a much greater decline while the value of λ increases from 0.0004 to 0.04. It can be interpreted as that when the stifling rate λ is stronger, more spreaders are likely to cease spread of the rumor and become stiflers, which results in smaller number of spreaders. It should be noted that the increase of the stifling rate λ primarily depends on improving the credibility of the institutional mainstream media. One possible interpretation for this is that, under crisis situations, if the institutional mainstream media have a lower credibility, then we can surmise that people may tend to trust information they receive from their acquaintances instead of trusting institutional mainstream media, and their collective information processing is very likely to encourage rumors.

Fig. 6 demonstrates how size of the spreader class $S(t)$ changes over time under different cooling off rates δ_1. As shown in Fig. 6, the number of spreaders shows a much greater decline while the value of δ_1 increases from 0.006 to 0.6. It should be noted that the increase of the cooling off rates δ_1 primarily depends on setting up prompt response systems to refute the incorrect information and cultivating the public's strong sense of social responsibility. One possible interpretation for this is that, if unambiguous and reliable information is not provided to the affected area in a timely manner, their collective information processing is very likely to encourage rumors. It can be inferred from this logic that, provision of timely and certain information may lead to successful crisis management. Considering the "2011 Xiangshui chemical explosion

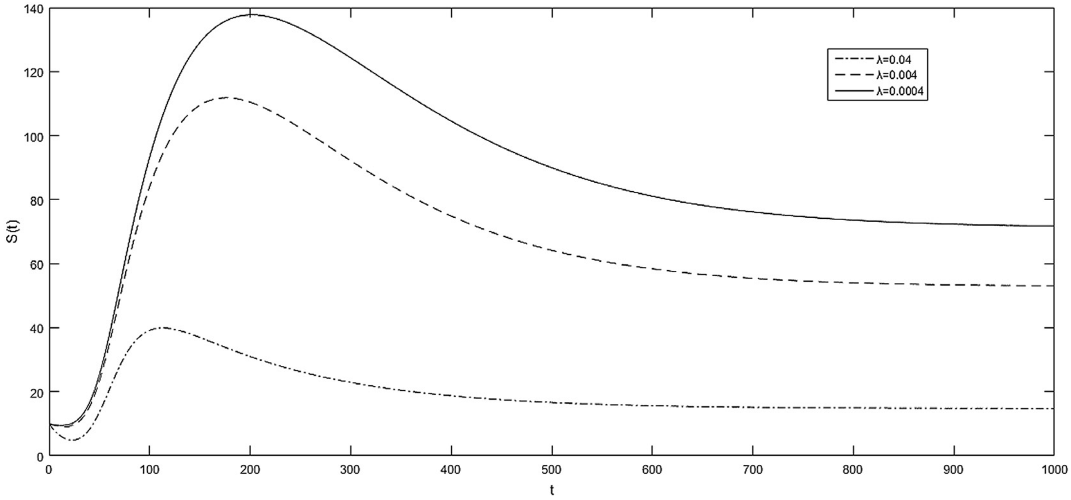

Fig. 5. Numbers of spreaders $S(t)$ versus time varying over different stifling rate λ with $\varepsilon = 2$, $\gamma_1 = 0.0001$, $\gamma_2 = 0.00001$, $\rho = 0.004$, $\alpha = 0.003$, $\delta_1 = 0.006$, $\delta_2 = 0.02$.

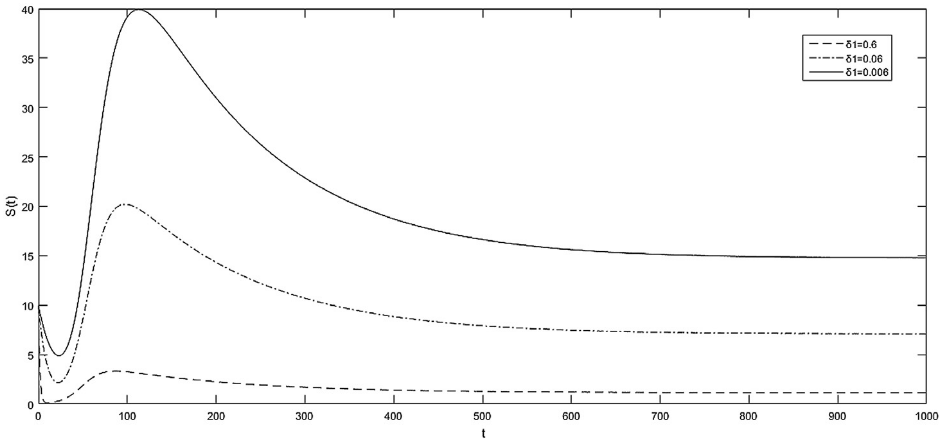

Fig. 6. Numbers of spreaders $S(t)$ versus time varying over different cooling off rate δ_1 with $\varepsilon = 2$, $\gamma_1 = 0.0001$, $\gamma_2 = 0.00001$, $\rho = 0.004$, $\alpha = 0.003$, $\lambda = 0.04$, $\delta_2 = 0.02$.

rumor'' event, the crisis management department workers went to the chemical industrial park for a field survey, they told the local residents that there was no chemical explosion. Shortly after, many residents followed the advice of the crisis management department and returned home. In addition, freedom and responsibility are inseparable in the field of information transmission. As a result, citizens should shoulder corresponding social liabilities when spreading information and express their views on social media within a scope allowed for by the law. It is worth mentioning that, after an earthquake jolted Jiuzhaigou, in Southwest China's Sichuan Province, in early August 2017, some people spread rumors in the name of the local earthquake watchdog that there would be strong aftershocks in the region, which naturally aroused huge panic among residents in the quake-hit area. Therefore, citizens should bear in mind that greater freedom of speech is guaranteed by greater responsibility.

5. Conclusion and future research

By extending the traditional rumor theory to the crisis management context, we propose an ILSCR rumor spreading model to discuss the control of rumor spreading in emergencies. The numerical results indicate that enhancing the public's capacity of identifying rumors, improving the credibility of the institutional mainstream media, setting up prompt response systems to refute the incorrect information and cultivating the public's strong sense of social responsibility may be effective rumor control measures.

Several limitations to our study should be noted. This study is an application of nonlinear dynamics theory to an emergency triggered by a chemical explosion rumor. Our suggested model needs replication and refinement in different emergencies contexts. Further, as we assume that the rumor spreads in a population with constant emigration, there could be information loss during analysis. To overcome this limitation, given that the flow of people is random in real life, it is more reasonable to take into account different emigration rates of different classes in future research. Moreover, this study does not consider the influence of the network structure for the time being, we will consider the impact of network structure on rumor spreading in our future research.

We suggest two promising research opportunities: First, this study could be further extended with future research by analyzing the social media data on emergencies. In the past, the study of rumor spreading has been hampered by the lack of social media data from emergencies. However, the introduction of Wechat and other social media services has provided researchers with a precious window of data on emergency information, usually in the immediate aftermath of emergencies. In this regard, analysis of social media data on emergencies will offer invaluable insight to explore the inherent laws of crisis management. Second, evidence from prior rumor events may be used to guide interventions and emergency response during emergencies. Therefore, the combination of archival data of social media and survey response data of the public who experienced the emergencies should be considered in future research.

CRediT authorship contribution statement

Guanghua Chen: Conceptualization, Methodology, Software, Data curation, Writing — original draft, Investigation, Software, Validation, Writing — reviewing & editing, Supervision.

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